# A Simple Method for Spacing the Adjacent Passbands of a Coupled-Line Filter

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*Abstract*—**A bandpass filter using two coupled transmission lines is considered. Two ports of the coupled-line four port are both short-circuited (or open-circuited); the other two ports are terminated in the characteristic impedances of the lines (e.g., 50 Ω). For a constant coupling along the coupling section $l$ the attenuation poles are fixed at $l/\lambda = n/2$ ($n = 0,1,2,\cdots$). If, however, the coupling changes along the coupling section in three steps with the coupling factors $k_1, k_2, k_3 = k_1$, the stopband between two adjacent passbands can be extended for certain values of $k_1$ and $k_2$. A simple calculation method for the coupling factors $k_1$ and $k_2$ is described. A practical design example shows good agreement with theoretical results.**

## I. Introduction

In the frequency range above 100 MHz, filter networks of coupled and uncoupled lines are used for a variety of applications. If confined to commensurate-length transmission-line elements, the design of such transmission-line filters may, by means of the "Richards" transformation, be based on the known design methods of lumped-element reactance networks. An extensive compilation of the various transmission-line filter types, structural details, and design methods together with a very detailed bibliography is given in [1]–[4] and constructional details are to be found in [5]–[6]. In the filters described in the literature, the use of coupled lines is almost exclusively limited to lines with constant coupling; exceptions are just the stepped digital elliptic filter by Rhodes [1] and some all passes, which are mostly used as phase shifters and delay lines. The following will analyze the effect of the coupling in a bandpass using two stepped coupled lines.

## II. Transmission Characteristics for Constant Coupling

A filter using two coupled lines, as shown in Fig. 1, is considered. The two lines have the characteristic impedance $Z_0$ and are electromagnetically coupled along a coupling section of the length $l$. The two ports 3 and 4 are short-circuited, while ports 1 and 2 are match terminated with $Z_0$.

The line coupling is characterized by the coupling factor $k$, which is defined by [1] the even- and odd-mode impedances $Z_{oe}$ and $Z_{oo}$, respectively:

$$k = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}, \qquad Z_{oe} \cdot Z_{oo} = Z_o^2. \tag{1}$$

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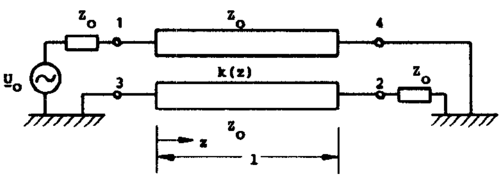


Fig. 1. Setup of the coupled-line filter.

The squared magnitude $S_{12}^2$ of the complex transmittance (scattering factor) $S_{12}$ of the two-port according to Fig. 1 is calculated from the known four-port equations of two coupled lines [1] and the port conditions to be

$$S_{12}^2 = \frac{4 \cdot k_0^2 \cdot (1 - k_0^2) \cdot \sin^2 \Theta}{(1 - k_0^2 \cdot \cos^2 \Theta)^2}, \qquad \Theta = 2\pi \frac{l}{\lambda}. \tag{2}$$

Analyzing (2) shows that the magnitude of the transmittance is periodic in $l/\lambda$ and has zeros and extrema. The zeros are lying at $l/\lambda = n/2$ ($n = 0,1,2,\cdots$), independent of the coupling factor. The number and the magnitude of the extrema are, however, very much dependent on $k_0$. In the range $0 \leqslant k_0 \leqslant 0.707$ *one* maximum of the transmittance occurs having the value

$$S_{12\max} = 2 \cdot k_0 \cdot \sqrt{1 - k_0^2}, \qquad \text{for } 0 \leqslant k_0 \leqslant 0.707.$$

For coupling factors $0.707 < k_0 < 1$ the transmittance has *two* maxima and *one* minimum (equiripple behavior) with the values:

$$\left.\begin{aligned} S_{12\max} &= 1 \\ S_{12\min} &= 2 \cdot k_0 \cdot \sqrt{1 - k_0^2} \end{aligned}\right\}, \qquad \text{for } 0.707 < k_0 < 1.$$

In the bandpass according to Fig. 1, maximum values with $S_{12} = 1$ (attenuation zeros) can only be achieved for coupling factors $k_0 \geqslant 0.707$. Fig. 2, as an example of some coupling factors $k_0$, presents the magnitude of the transmittance as a function of $l/\lambda$; for equal-ripple behavior a maximum passband attenuation of 0.5 dB has been assumed, which corresponds to a coupling factor of $k_0 = 0.815$.

## III. Transmission Characteristics for Stepped Coupling

### A. Transmittance

As shown in Fig. 2, the stopband between the first and the second passband at $l/\lambda = 0.5$ is narrow, which in

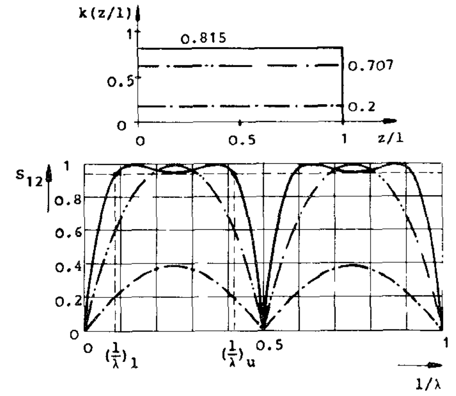


Fig. 2. Characteristic of the transmittance $S_{12}$ for constant coupling.

practical operation is often troublesome. In the following design, the stopband between two passbands is extended by the use of simple measures.

By contrast to the description in Section II, the filter according to Fig. 1 is now investigated with a variable coupling distribution. The coupling along the coupling section varies in three steps of equal length with the coupling factors $k_1, k_2, k_3 = k_1$ and is, as is depicted in Fig. 3, symmetrical to the center of the coupling section at $z = l/2$. For each individual step, the relationship between the coupling factor and the even- and odd-mode impedance is given by (1).

The transmittance $\boldsymbol{S}_{12}$ of the two-port of stepped coupled lines (Fig. 1) is calculated analogous to Section II. from the four-port relations and port conditions, whereby it is convenient to use the calculation methods for the scattering matrix ($\boldsymbol{S}$) of directional couplers [9]. Finally, one obtains the squared magnitude $S_{12}^2$ of the complex transmittance $\boldsymbol{S}_{12}$ to be

$$S_{12}^2 = \frac{4 \cdot P^2}{(1+P^2)^2} \tag{3}$$

with

$$P = \sin\Theta \cdot (A_1 - \sin^2\Theta \cdot (A_1 + A_2))$$
$$A_1 = \frac{2k_1}{W_1} + \frac{k_2}{W_2}, \qquad W_i = \sqrt{1-k_i^2}$$
$$A_2 = \frac{2k_1 - k_2 \cdot (1+k_1^2)}{W_1^2 \cdot W_2}, \qquad \Theta = 2\pi \frac{l}{3\lambda}. \tag{4}$$

From (3) it is easy to determine the conditions for attenuation poles and zeros of the bandpass of two stepped coupled lines. The occurrence of attenuation zeros is subject to the condition $S_{12} = 1$, which is only satisfied using (3) if $P^2 = 1$. Attenuation poles are given by the zeros of $P$.

As seen from (4), $P$ is an odd real polynomial of the third degree in the frequency variable $\sin\Theta$. The zeros of $S_{12}$ and/or $P$ are lying at $l/\lambda = 3n/2$ ($n = 0, 1, 2, \cdots$) and

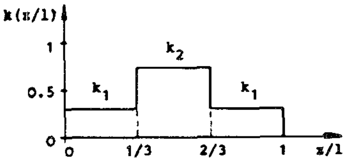


Fig. 3. Characteristic of the symmetrical stepped coupling factor.

those dependent on the coupling factors at

$$\frac{l}{\lambda} = \frac{3}{2}\left\{ n \pm \frac{1}{\pi} \arctan \sqrt{\frac{2 \cdot k_1 \cdot W_1 \cdot W_2 + k_2 \cdot (1-k_1^2)}{2 \cdot k_1 - k_2 \cdot (1+k_1^2)}} \right\}. \tag{5}$$

Since physically realizable values of the coupling factor are between 0 and 1 according to (1), the numerator of the root argument in (5) is always greater than zero. Depending on the values of the coupling factors $k_1$ and $k_2$, the denominator can, however, be both greater and less than zero. Hence according to (5), additional attenuation poles can only be obtained for $2k_1 - k_2 \cdot (1+k_1^2) \geqslant 0$. For the special case $2k_1 = k_2 \cdot (1+k_1^2)$ these poles coincide in one double pole at $l/\lambda = 3 \cdot (2n+1)/4$, while for $2k_1 > k_2 \cdot (1+k_1^2)$ two poles are symmetrical to $l/\lambda = 3 \cdot (2n+1)/4$. Consequently, a stopband that is as broad as possible can only be achieved for $2k_1 > k_2 \cdot (1+k_1^2)$. The following paragraph will deal with the calculation of the coupling factors $k_1$ and $k_2$, taking into account certain attenuation requirements.

## B. Calculation of the Coupling Factors

If the polynomial $P$ in (4) is known, the squared magnitude (3) of the transmittance is fixed. Hence the polynomial $P$ must be determined such that the attenuation requirements for the passband and stopband are met, taking care that $P$ only includes the unknown coupling factors $k_1$ and $k_2$.

For the further analysis it is convenient to introduce the abbreviation $u = \sin\Theta$, with $\Theta$ being given by (4). Then the polynomial $P$ reads

$$P(u) = u \cdot (A_1 - u^2 \cdot (A_1 + A_2)) = u \cdot G(u) \tag{6}$$

with the even real polynomial $G(u)$.

The zero $u_{01} = -u_{02}$ of $G$ must be real and because of $0 \leqslant |u| \leqslant 1$ must be inside of the interval $0 < u \leqslant 1$, if additional attenuation poles are required. Since $P(u)$ is odd, the derivation $P'(u)$ is even, it follows that $P(u)$ also has an extremum (zero of $P'(u)$) in this interval. In order to determine a passband using this maximum, there must be $P_{max} \geqslant 1$ on this location, according to (3).

From this consideration the characteristic as shown in Fig. 4 is derived for the odd real polynomial $P(u)$, with the associated attenuation characteristic being calculated according to (3).

On the location $u = u_1$, the polynomial $P(u)$ has the value $P(u_1) = 1 + d_1$, while the derivation $P'(u_1) = 0$. At

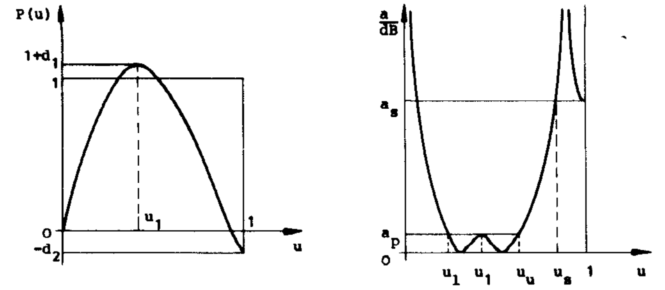


Fig. 4. Characteristic of the polynomial $P$ as a function of $u=\sin\Theta$ and the associated attenuation.

$u=1$ there is $P(1)=-d_2$. The constants $d_1$ and $d_2$ are calculated by means of (3) from the prescribed attenuation values in the passband and stopband $a_p$ and $a_s$ (see Fig. 4) to be

$$d_1=\left(m+\sqrt{m\cdot(2-m)}\right)/(1-m), \qquad m=1-10^{-(a_p/20)}$$

$$d_2=\left(1-\sqrt{1-n^2}\right)/n, \qquad n=10^{-(a_s/20)} \tag{7}$$

with $a_{p/s}$ to be indicated in decibels. Since only two unknown coupling factors $k_1$ and $k_2$ are to be determined, $u_1$ cannot be prescribed arbitrarily.

With the above three constraints for $P(u)$ the following equations are obtained:

$u=1$

$$P(1)=-d_2=A_1-(A_1+A_2), \qquad A_2=d_2 \tag{8}$$

$u=u_1$

$$P(u_1)=1+d_1=u_1\cdot\left(A_1-u_1^2\cdot(A_1+A_2)\right) \tag{9}$$

$$P'(u_1)=0=A_1-3u_1^2\cdot(A_1+A_2), \qquad u_1^2=\frac{A_1}{3\cdot(A_1+A_2)}. \tag{10}$$

Substituting (8) and (10) into (9) gives

$$A_1^3-\frac{27}{4}\cdot(1+d_1)^2\cdot A_1-\frac{27}{4}\cdot(1+d_1)^2\cdot d_2=0. \tag{11}$$

Taking into account (4), the unknown coupling factors $k_1$ and $k_2$ can be calculated from (8) and (11). The nonlinear equation system

$$f_1=A_1^3(k_1,k_2)-\frac{27}{4}\cdot(1+d_1)^2\cdot A_1(k_1,k_2)-\frac{27}{4}\cdot d_2\cdot(1+d_1)^2=0$$

$$f_2=A_2(k_1,k_2)-d_2=0 \tag{12}$$

is solved by means of iteration procedures [8].

### C. Examples

As a first example a bandpass with maximally flat passband (i.e., in (7) $d_1=0$) and a stopband attenuation of $a_s=20$ dB is considered. The values of the unknown coupling factors $k_1$ and $k_2$, necessary to fulfill the attenuation requirements given above, are calculated from (12) using the "Newton-Raphson" iteration procedure [8] to $k_1=0.5283$ and $k_2=0.8094$. The lower and upper band limit of the passband $(l/\lambda)_l$ and $(l/\lambda)_u$, defined at the attenuation value $a_{pl/u}=3$ dB, and the stopband limit $(l/\lambda)_s$ are calculated iteratively from (6) to $(l/\lambda)_l=0.092$; $(l/\lambda)_u=0.525$; $(l/\lambda)_s=0.658$. The attenuation and the coupling characteristic for this filter is presented in Fig. 5.

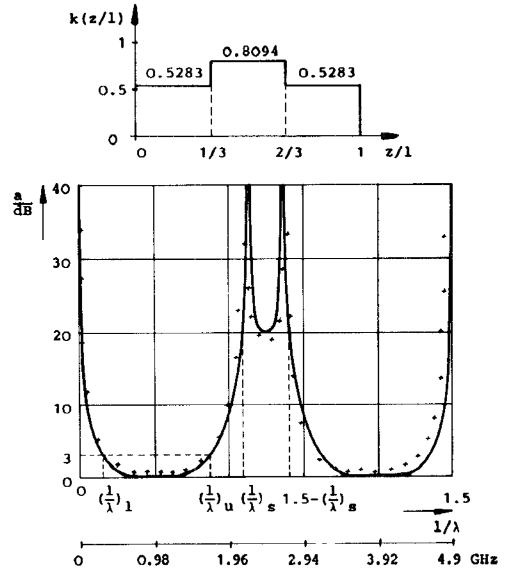


Fig. 5. Attenuation characteristic of the filter with maximally flat passband and a minimum stopband attenuation of $a_s=20$ dB.—— = theoretical; + = measured.

The second filter considered has an equal-ripple passband behavior with a ripple of $a_p=0.3$ dB and a stopband attenuation of $a_s=60$ dB. The unknown coupling factors and band limits can be calculated in the same way given in the first example. The values obtained are: $k_1=0.601$; $k_2=0.883$; $(l/\lambda)_l=0.116$; $(l/\lambda)_u=0.492$; $(l/\lambda)_s=0.741$. Fig. 6 shows the attenuation characteristic of this filter.

## IV. Experimental Results

To verify the theoretical calculations by experiments some filters were built. The manufacture of the filters using stepped coupled lines was based on the experience, findings and the technological know-how acquired for directional couplers using stepped coupled lines [5], [6], [9]. The experimental investigations have shown, that a very suitable realization appears to be in rectangular shielded striplines, since it is easy to attach mechanisms for fine-tuning of the filter [10].

In Fig. 7 the configuration of a constructed filter is outlined. The characteristic impedances of the lines are $Z_0=50\ \Omega$. To achieve a filter response according to Fig. 5, the design of the coupled lines was based on the coupling factor values $k_1=0.5283, k_2=0.8094$. The length of the coupling section was chosen to be $l=92$ mm according to a lower 3-dB passband frequency $f_l=300$ MHz. For reasons of convenient realization the ratio $t/b$ of the line

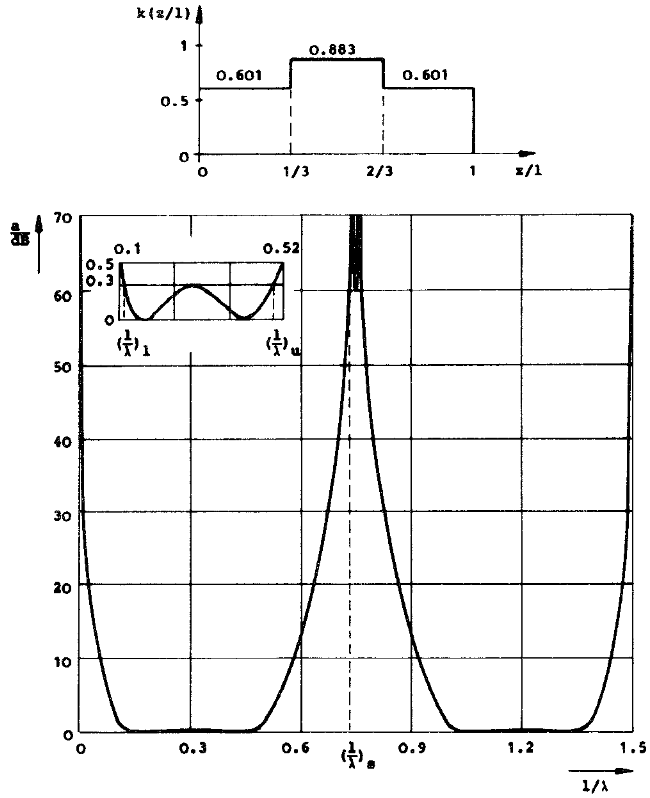


Fig. 6. Attenuation characteristic of the filter with equiripple passband, $a_p = 0.3$ dB, and a minimum stopband attenuation of $a_s = 60$ dB.

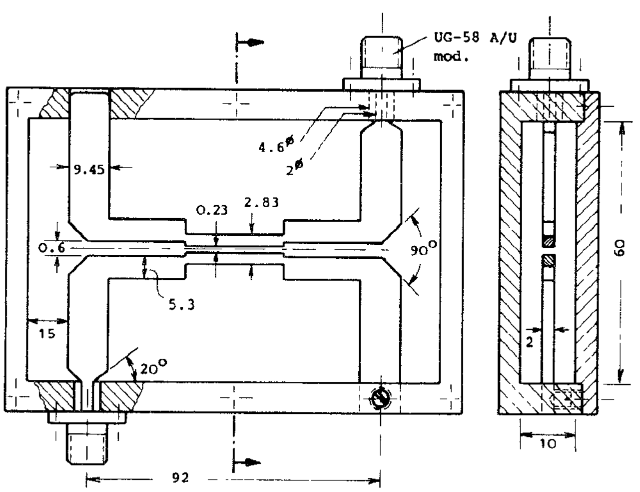


Fig. 7. Drawing of a manufactured filter (values given in millimeters).

thickness $t$ to the ground plane spacing $b$ was chosen to be $t/b = 0.2$ with $b = 10$ mm. Modified UG-58 A/U type-N connectors have been used. The material of the filter is aluminum. Fig. 7 shows additional construction details and additional dimensions.

The detailed design of the resonator ends and of the filter input/output has been performed under the considerations to achieve 1) low VSWR in the passband, 2) low cross coupling between resonators and input/output section, and 3) mechanical stable configuration of the coupled lines. Fig. 7 shows the result. The junction between connector and 50-Ω single stripline is performed by a coaxial line with the center conductor diameter $d = t$, whereby the stripline is tapered [5] with an angle of 20° to give a low-reflection transition. According to Fig. 7, the junction between single stripline and coupled line is constructed as a 90° bend with a 45° slanted stripline edge to avoid cross coupling and to save a soft outcoupling. To realize the short-circuit and to ensure mechanical stability, the other line end is fixed in a housing groove by a screw. In Fig. 5, the measured attenuation characteristic of the filter is shown. The minimum attenuation in the passband is 0.63 dB.

## V. Conclusion

The parametric investigations of the filter according to Fig. 1 have shown that the transmission characteristics of this filter can be substantially changed by a stepped coupling between the two lines. This implies the possibility to introduce additional degrees of freedom in the optimization of transmission-line filters. As a result, this optimization can be improved to a certain extent, as has been demonstrated by preliminary calculations. It remains now for further investigations to determine the effect of the terminating resistances and to prove whether these filters can be included in the synthesis of conventional transmission-line filters. Concerning the filter realization in printed stripline technique [6], more detailed investigations are necessary to find the optimum design.

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